**Unit 2**

**Operations** with rational numbers

Objectives:

* to compare and order rational numbers
* to solve problems involving operations on rational numbers

**Lesson 1 – Types of Numbers: Natural, Whole, Integers, Rational, Irrational and Real**

**Natural Numbers ( ):**

Natural numbers are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ numbers

= {1, 2, 3, 4, 5, 6, 7, …}

**Whole Numbers ( ):**

Whole number are the natural number \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ zero

= {0, 1, 2, 3, 4, 5, 6, …}

**Positive Numbers:**

Positive number are 1, 2, 3, 4, 5, …

Positive numbers: {1, 2, 3, 4, 5, …}

**Negative Numbers**

Negative numbers are … , -5, -4, -3, -2, -1.

Negative numbers: {…, -5, -4, -3, -2, -1}

**Integers ( ):**

Integers are all of the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ numbers together with all of the \_\_\_\_\_\_\_\_\_\_\_\_ numbers

Integers are the set containing the positive numbers, the negative number and zero

Zero is \_\_\_\_\_\_\_\_\_\_\_\_\_ positive nor negative.

In other words, integers are defined as the set of whole numbers and their \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

={…, -3, -2, -1, 0, 1, 2, 3, …}

**Rational Numbers ( ):**

A Rational Number is any number that can be written in the form \_\_\_\_\_\_\_\_\_\_\_ or () where and are both integers and \_\_\_\_\_\_ is not \_\_\_\_\_\_\_\_\_\_.

Rational numbers include all \_\_\_\_\_\_\_\_\_\_\_

* Proper Fractions: numbers with the numerator smaller than the denominator

Eg:

* Improper Fractions: numbers with the numerator larger than the denominator

Eg:

* Mixed Numbers:

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ roots may be rational numbers if their standard form is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ number

In rational numbers, the denominator \_\_\_\_\_\_\_\_\_\_\_\_\_\_ be non-zero.

Examples of Rational Numbers ( ):

* 2 can be expressed in the form as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_ or \_\_\_\_\_\_ or \_\_\_\_\_\_ since all \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ all are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (repeats)
* \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (repeats)

**Irrational Numbers:**

Irrational numbers are all number which \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ be expressed as a ratio of integers.

As decimals, they never \_\_\_\_\_\_\_\_\_\_\_\_\_\_ or \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

They go on forever, or \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Examples of irrational numbers:

* Irrational (never \_\_\_\_\_\_\_\_\_\_\_\_\_ or \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)
* Irrational (never \_\_\_\_\_\_\_\_\_\_\_\_\_ or \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)
* 1.23223222322223222223… Even though it has a pattern, it does not repeat or terminate and therefore it is Irrational

**Real Numbers ( ):**

Real number are \_\_\_\_\_\_\_\_\_\_ numbers, irrational or rational.

Any number that you can find on the \_\_\_\_\_\_\_\_\_\_\_\_\_\_ line is real.

A real number is a number used to name the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of a given point from 0.

All Natural numbers are Whole numbers, which are Integers, which are Rational numbers which are Real number.

All irrational number are \_\_\_\_\_\_\_\_\_\_ numbers, but not all Real numbers are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Homework:

**Lesson 2 – Understanding rational numbers**

Rational numbers are \_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_ numbers that can be written in \_\_\_\_\_\_\_\_\_\_\_\_ form.

Examples of rational numbers:

Examples of irrational numbers:

**Example 1**: Reduce to lowest terms

(a) (b) (c) (d) (e)

* To write a rational number in decimal form, \_\_\_\_\_\_\_\_\_\_ the top number (\_\_\_\_\_\_\_\_\_\_\_\_\_\_) by the bottom number (\_\_\_\_\_\_\_\_\_\_\_\_\_\_).

**Example 2**: Write in decimal form

(a) (b) (c)

= = =

(d) (e)

= =

* To change a decimal number to fraction form, write the number without the decimal in the numerator and write the place value of the number in the denominator. Then reduce the fraction if possible.

**Example 3**: Write in fraction form

(a) 0.23 is read as 23 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ =

(b) 0.075 is read as 75 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_=

The number of zeros in the denominator corresponds to the number of \_\_\_\_\_\_\_\_\_\_ after the decimal.

**Example 4:** Write each rational as an equivalent fraction.

(a)  (b)  (c) 

**Example 5:** Compare and order the following rational numbers. Write them in ascending order and descending order.

 -0.6   -1

**Example 6:**

(a) Which fraction is greater?  or 

(b) Which fraction is smaller? **** or ****

**Example 7:** Identify a fraction between each pair of rational numbers:

(a) -0.6 and -0.7 (b) -2.4 and -2.5

**Example 8:** Identify a decimal between each pair of rational numbers:

(a)  (b) 

**Example 9:** Replace each □ with >, <, or = to make each statement true.

(a)  (b) 

(c)  (d) 

(e)  (f) 

*Homework:*

**Lesson 3 – Problem Solving with Rational Numbers in Decimal Form Part 1(2.2)**

Integer operations must be reviewed before we begin to solve problems with rational problems.

Integers are \_\_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_ numbers on the number line.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

* Adding and Subtracting Integers

Refer to the following example: -3 + 7

When adding numbers on the number line begin with the first number given, -3, and move 7 units to the \_\_\_\_\_\_\_\_\_\_\_\_.

Therefore, -3 + 7 = \_\_\_\_.

Refer to the following example: 4 – 8

When subtracting numbers on the number line begin with the first number given, \_\_\_\_, and move \_\_\_\_ units to the \_\_\_\_\_\_\_\_\_\_\_\_.

Try these:

1. -3 – 5 = \_\_\_\_ 2. -1 + 8 = \_\_\_\_ 3. –9 + 6 = \_\_\_\_
2. 5 + 4 = \_\_\_\_ 5. 10 – 15 = \_\_\_\_ 6. -2 – 7 = \_\_\_\_

Sometimes the numbers we add and subtract are too big for us to move right or left on the number line. Therefore, we use the following rule to add or subtract integers:

If the signs in the question are the \_\_\_\_\_\_\_\_ we add the numbers and the answer takes the sign of the \_\_\_\_\_\_\_\_\_\_ number.

Example: (a) -12 – 20 = \_\_\_\_ (b) 45 + 21 = \_\_\_\_

If the signs in the question are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ we subtract the numbers and the answer takes the sign of the \_\_\_\_\_\_\_\_\_\_\_\_ number.

Example: (a) -25 + 50 = \_\_\_\_ (b) 13 – 33 = \_\_\_\_

(c) -40 + 10 = \_\_\_\_ (d) 100 – 65 = \_\_\_\_

Sometimes when adding or subtracting integers there are more than one sign between the numbers. When this occurs, you must change the signs in the middle to only \_\_\_\_\_\_ sign and use the above rules learned.

If the signs in the middle are the \_\_\_\_\_\_\_\_\_\_\_\_, change the signs to a single \_\_\_\_\_\_\_\_\_\_ sign.

If the signs in the middle are \_\_\_\_\_\_\_\_\_\_\_\_, change the signs to a single \_\_\_\_\_\_\_\_\_\_ sign.

Example: (a) 3 – (+12) = \_\_\_\_\_\_\_\_\_ = \_\_\_ (b) -4 – (-7) = \_\_\_\_\_\_\_\_\_ = \_\_\_

(c) 15 + (-25) = \_\_\_\_\_\_\_\_\_ = \_\_\_ (d) 6 – (-9) = \_\_\_\_\_\_\_\_\_ = \_\_\_

Now we will work with decimal numbers.

**Practice:** Estimate and calculate

(a) (b)

(c) (d)

**Example 1:** Estimate and calculate.

(a) (b)

(c) (d)

**Example 2**: Complete each statement

1. □ + 1.8 = -3.5 (b) -13.3 - □ = -8.8

Homework:

**Lesson 4 – Problem Solving with Rational Numbers in Decimal Form Part 2(2.2)**

* Multiplying and Dividing Integers

When multiplying and dividing integers the only rule to remember is to count the number of \_\_\_\_\_\_\_\_\_\_\_\_ signs and if it is an even number then you have a \_\_\_\_\_\_\_\_\_\_\_\_ answer and if it is an odd number of negative signs then you have a \_\_\_\_\_\_\_\_\_\_\_\_ answer. Once you know the sign, multiply or divide as normal.

Example: (a) = \_\_\_\_ (b) = \_\_\_\_ (c) 3 × -7 = \_\_\_\_

(d) -9 × -9 = \_\_\_\_ (e) (-1)(-1)(6)(-10) = \_\_\_\_ (f) (-2)(-1)(-1)(-4) = \_\_\_\_

Now we will work with decimal numbers.

**Practice:** Estimate and calculate

(a) (b)

(c) (d)

(e) (f)

**Example 1:** Estimate and calculate.

(a) (b)

**Example 2:** Calculate, showing all steps. (Remember Order of Operations)

(a) (b)

(c) (d)

**Example 3**: Complete each statement

1. □ × (-4.5) = -9.45 (b) -18.5 ÷ □ = 7.4

**Example 4**: On Saturday, the temperature at the Blood Reserve near Stand Off, Alberta decreased by 1.2 C°/h for 3.5 h. It then decreased by 0.9 C°/h for 1.5 h.

1. What was the total decrease in temperature?
2. What was the average rate of decrease in temperature?

**Example 5:** A hot-air balloon climbed at 0.8 m/s for 10s. It then decreased at 0.6 m/s for 6s.

1. What was the overall change in altitude?
2. What was the average rate of change in altitude?

*Homework:*

**Warm-up: Review of Fractions**

* Adding and subtracting fractions with a common denominator: add/subtract the numerators then reduce, if possible.
* Adding/subtracting fractions with different denominators: obtain the same denominator by finding lowest common multiple (LCM), **or** by cross multiplying, then reduce, if possible.
* Multiplying fractions: multiply their numerators and multiply the denominators then reduce, if possible; **or** by cross-cancelling.
* Dividing Fractions: Flip the second and multiply; reduce if possible.

Note: When working with mixed fractions you must change them to improper fractions before performing the operation.

**Lesson 5 – Problem Solving with Rational Numbers in Fraction Form (Part 1) (2.3)**

**Example 1**: Calculate

(a)  (b) 

(c)  (d) 

(e)  (f) 

(g)  (h) 

**Example 2**: Complete each statement.

(a)  (b) 

(c)  (d) 

Homework:

**Lesson 6 – Problem Solving with Rational Numbers in Fraction Form (Part 2) (2.3)**

Order of Operations involving Fractions:

**Example 1**: Calculate

(a) (b)

(c) (d)

**Example 2**: At the start of a week, Maka had $30 of her monthly allowance left. That week, she spent of the money on bus fares, another  shopping and  on snacks. How much did she have left at the end of the week?

**Example 3**: Stephen had $46 in a bank account that he was not using. Each month for three months, the bank withdrew  of the amount still in his account as a service fee. How much was left in the account after the last withdrawal?

*Homework*:

**Lesson 6 – Determining Square Roots of Rational Numbers (2 .4) (optional)**

*Recall*: When we square a number, we multiply it by itself, i.e. 72 = 7×7 = 49

Since 7×7 = 49, we say that 7 is the square root of 49, and we write.

All *positive* numbers have square roots. Those that have square roots which can be expressed as the product of two equal rational factors are called *perfect squares*.

i.e. 25 is a perfect square because 25 = 5 × 5,

 is a perfect square because  = 

30 is NOT a perfect square, therefore 30 is referred as a non-perfect square root.

We can estimate the square roots of numbers that are not perfect squares.

**Example 1**: Estimate and calculate the area of a square photo with a side length of 7.1 cm.

*How to determine if a number is a perfect square?*

* The square root of the number must be a whole number
* When taking the square root of fractions, square root the numerator and the denominator *separately*.
* When taking the square root of decimals, change the decimal to a fraction then square root

**Example 2**: Determine whether each of the following numbers is a perfect square.

(a)  (b) 0.4 (c)  (d) 1.2 (e) 0.09

*Note:* Decimal numbers that are perfect squares have *even* decimal places.

**Example 3**: Evaluate without a calculator and then verify your answers with a calculator.

* Move the decimal to the right until you get a whole number (note the number of spaces you moved the decimal)
* Take the square root
* Move the decimal back to *half* the number of spaces to the left

*or*

* Write the number as a fraction
* Take the square root of the numerator and the square root of the denominator

(a)  (b)  (c)  (d) √.0121

**Example 4:** Estimate and calculate.

*Steps to estimating*:

* Move the decimal to the right until you get a whole number (note the number of spaces you moved the decimal)
* Determine the perfect square before and after the number
* Approximate the square root (it may be helpful to draw a number line)

(a)  (b) 

Homework: